Viscous dissipation effects on thermophoretically augmented aerosol particle transport across laminar boundary layers

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The sensitivity of aerosol particle motion to local temperature gradients has motivated this investigation of viscous dissipation effects on mass transport rates across nonisothermal, low mass-loading 'dusty gas' laminar boundary layers (lbl). From numerical lbl transfer calculations, including 'ash' particle thermophoresis and variable thermophysical properties, it has been found that for a specified wall temperature, T_w, and mainstream static temperature, T_e, viscous dissipation within the boundary layer increases total particle deposition rates, its relative importance being dependent on T_w/T_e. For combustion turbine blades which operate at near-unity Mach number, neglect of viscous dissipation is found to cause about a 25% underestimate of the fouling rate at $T_w/T_e = 0.8$ for particle diameters between $0.6 \times 10^{-2} \mu m$ and $0.3 \mu m$. Alternatively, for conditions of fixed adiabatic wall temperature, T_{aw}, or fixed stagnation (reservoir) temperature, T_0, dusty gas acceleration to appreciable Mach numbers is associated with reduced particle arrival rates due, in part, to the associated reduction in mainstream gas temperature. Recently developed mass transfer rate correlations are extended and found to be successful when tested against the present numerical calculations.

Keywords: *laminar boundary layers, thermophoresis, viscous dissipation, fouling*

Thermophoresis (the drift of particles down a temperature gradient) dramatically influences small particle ('heavy molecule') diffusional mass transfer rates from high temperature (e.g. combustion) gases, as shown experimentally in Ref 1, and treated theoretically in Ref 2. Since it is well known that viscous dissipation modifies temperature profiles in momentum diffusion boundary layers (bl)[3-7], the interaction of viscous dissipation with fly-ash particle thermophoresis merits further attention, especially in connection with the prediction of fouling rates in combustion turbines.

A mathematical formulation including viscous dissipation and/or wall transpiration (blowing or suction), and covering the particle size range from vapour molecules up to the usual onset of particle inertial effects was given for low mass-loading, ash-containing laminar forced convection bls with variable properties in Ref 2. Previously reported results, however, were restricted to thermophoretically augmented deposition rates to solid walls in the limit of negligible viscous dissipation. Here, the additional effects of viscous dissipation** for the flat plate geometry are demonstrated and agree with earlier results of Goren[10] in the limiting case of constant carrier gas and particle properties and large Schmidt numbers. Because of our present focus on low particle mass-loadings, the presence of aerosols does not modify the heat (and/or momentum) transfer problem; however, the conditions of convection heat transfer will be seen to modify deposition (e.g. fouling) rates.

Results and discussion

Current combustion turbine blades experience high subsonic Mach numbers[11-14]; therefore calculations emphasized this interval but covered the entire range $Ma_e \leqslant 2$. As in Ref 2, we numerically solved the coupled ordinary differential equations governing self-similar laminar boundary layer flow, using the method of finite-differences with a quasi-linearization scheme for treating the nonlinear terms. In Fig 1 are plotted the predicted thermophoretic enhancement in deposition rate versus particle size†† for the two temperature ratios: $T_w/T_e = 0.8$ and 0.6. The effect of viscous dissipation can be demonstrated by comparing the $Ma_e = 2$ curves with the corresponding $Ma_e = 0$ curves for the same gas stream static temperature, T_e. Note that the deposition rate augmentation for the case of $T_w/T_e = 0.8$, $Ma_e = 2$ is about the same as that of the case of $T_w/T_e = 0.6$, $Ma_e = 0$ for

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** The effect of transpiration cooling on thermophoretically-enhanced deposition rates is treated elsewhere for both laminar[8] and turbulent[9] forced convection bls

†† In Figs 1 and 2, g is the number of monomer units (Na_2SO_4) in the cluster

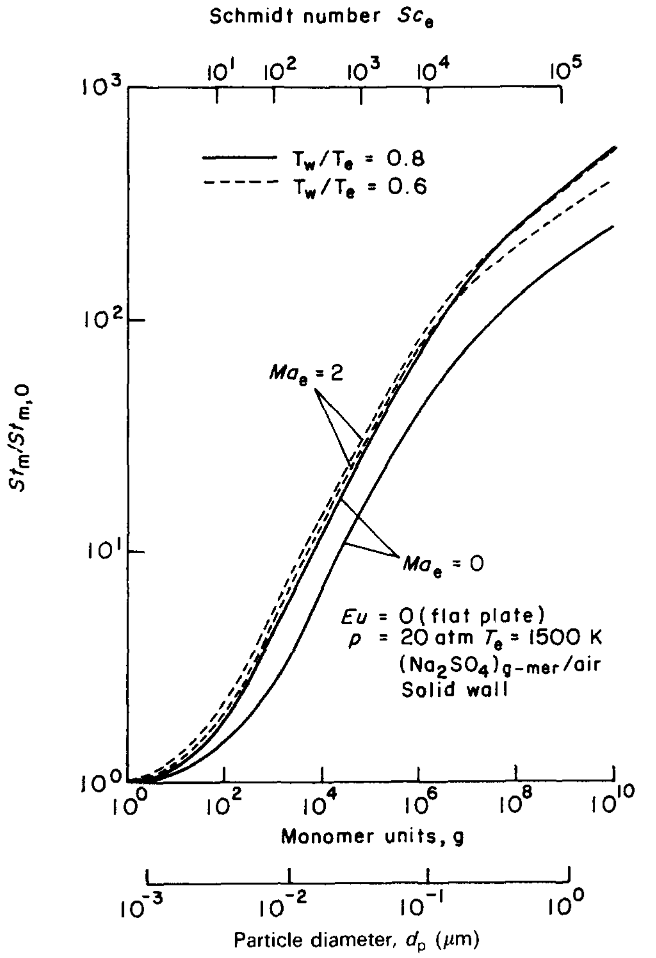

Fig 1 Effect of thermophoresis and viscous dissipation on mass transfer coefficient as a function of particle size in a laminar boundary layer at different temperature ratios

about a 0.1 μm diameter particle. However, the effect of viscous dissipation is dependent on the temperature ratio, T_w/T_e. As T_w/T_e departs from unity, increasing the importance of thermophoresis, the dissipation-induced modification of temperature profiles diminishes, decreasing the relative importance of viscous dissipation. The isolated effect of viscous dissipation on thermophoretically enhanced small particle deposition rates displayed in Fig. 2, exhibits a plateau for a wide range of particle sizes†. The plateaus indicate that for $Ma_e = 1$, viscous dissipation increases deposition rates by about 25% for $T_w/T_e = 0.8$, and by about 5% for $T_w/T_e = 0.6$. We conclude that, for current combustion turbine applications (with, say, $Ma_e = 0.8$), the additional effects of viscous dissipation on thermophretically augmented convective mass transfer are not very large, but neglecting them entirely would cause non-negligible systematic underestimations in predicted particle deposition rates.

As discussed in Refs 2 and 18, thermophoresis in effect introduces two phenomena into the boundary layer mass transfer process for cold solid walls: (1) the 'thermophoretic suction' effect associated with the thermophoretic velocity, V_T (proportional to the local temperature gradient), and (2) a 'thermophoretic pseudo-sink' effect associated with the thermophoretic pseudo-first-order 'reaction' rate constant, k (itself proportional to the spatial nonuniformity in ρV_T and, hence, dependent on the local curvature of the temperature profile; see below). Indeed, it is useful to think of the thermal bl as a plug-flow 'chemical' reactor of length δ_h through which there is a dusty gas flow with an average velocity $\overline{v+v_T}$, and a first-order reaction with an average rate constant $\bar{k}$.

† The deviation of the curves from near constant behavior is at particle diameters above about 0.3 μm, where particle inertial effects (not treated here) often set in[15-17]

Notation

$-B_T$	Thermophoretic suction parameter
d_p	Particle diameter
Eu	Euler number $= d \ln u_e / d \ln x$
g-mer	Cluster containing g monomer units ($g = 10^0, 10^1, \ldots, 10^{10}$)
k	Pseudo-first-order reaction rate constant associated with spatially variable V_T
Le	Lewis number = ratio of particle Brownian diffusivity to carrier gas heat diffusivity
Ma	Mach number
p	Pressure
Sc	Schmidt number = ratio of gas momentum diffusivity to particle Brownian (Fick) diffusivity
St_m	Stanton number for mass transfer
T	Absolute temperature
u	Fluid velocity in x-direction (parallel to wall)
V	Fluid velocity in y-direction (normal to wall)
V_T	Thermophoretic velocity
x	Distance along the surface
y	Distance normal to surface
α_T	Thermal diffusion factor of particle/gas combination
δ_h	Thermal boundary layer thickness
ρ	Density of mixture
ω	Mass fraction of particles in prevailing mixture ($\omega \ll 1$ throughout)

Subscripts

aw	Pertaining to the adiabatic wall condition
e	Outer edge of the boundary layer ('local mainstream')
m	Pertaining to the outer edge of the mass transfer (Brownian diffusion) bl
Ma_e	Pertaining to including viscous dissipation
w	At the surface (wall)
0	Pertaining to upstream 'stagnation' conditions (e.g. T_0) or to conditions without thermophoresis and viscous dissipation (e.g. $St_{m,0}$)

Superscripts

$\bar{}$	Averaged quantity (e.g. over the thermal bl)

Miscellaneous

bl	boundary layer (l ≡ laminar)

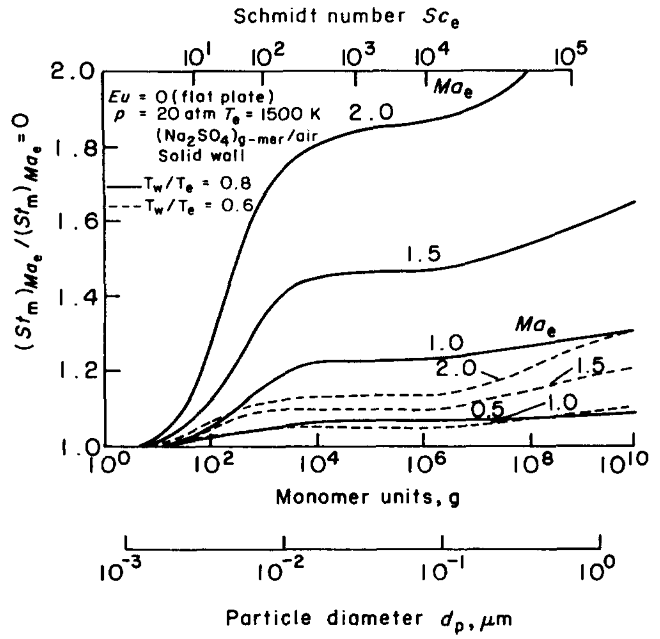

Fig 2 Effect of viscous dissipation on mass transfer coefficient as a function of particle size in a laminar boundary layer at different Mach numbers and temperature ratios

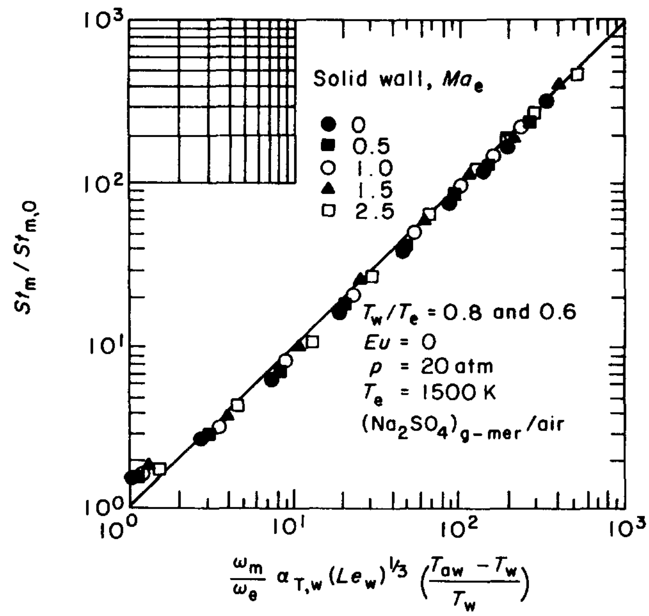

Fig 3 Correlation of the enhancement of mass transfer (deposition) rate with the product of thermophoretic 'sink' function and 'suction' parameter

When the particles reach the outer edge of the Brownian diffusion sublayer (ractically wall for $Sc \gg 1$), their mass fraction is reduced to ω_m (cf the mainstream value ω_e) by this 'reaction'. However, V_T and k are closely related through $k = -\rho^{-1}[\partial(\rho V_T)/\partial y]$. Therefore, any modification of the temperature profile due to viscous dissipation affects both the suction and pseudo-sink effects via V_T and k.

Based on the above-mentioned reasoning, rational engineering correlations have been developed to facilitate simple, yet accurate deposition rate predictions, including the augmentation due to thermophoresis in the presence of viscous dissipation and/or transpiration cooling[19,20]. For lbls with viscous dissipation we introduce a dimensionless thermophoretic suction parameter, $-B_T$, given by:

$$-B_T = \alpha_{T,w} Le_w^{1/3}[(T_{aw} - T_w)/T_w] \quad (1)$$

and describe the above-mentioned pseudo-sink effect via

$$\frac{\omega_m}{\omega_e} = \exp\left[-(\alpha_T Le)_e \frac{T_{aw} - T_w}{T_w}\right] \times \left(\frac{T_{aw}}{T_e}\right)^{-5(\alpha_T Le)_e(T_w/T_e)} \quad (2)$$

Then, for $-B_T > 5$, our mass transfer correlation reduces to the simple form:

$$St_m/St_{m,0} = (-B_T)(\omega_m/\omega_e) \qquad (\text{for } -B_T > 5) \quad (3)$$

In Eqs (1) and (2), α_T is the thermal diffusion factor of the particles and, in the limits of free molecule flow (relative to the particle) and continuum flow, is inversely proportional to the Brownian diffusion coefficient, i.e. α_T/Sc = constant. Both the thermal diffusion factor and Brownian diffusion coefficient are calculated from an interpolation between well known results applicable to each of the above-mentioned limits. Fig 3 demonstrates the success of Eq (3) when tested against our numerical calculations for a wide range of particle sizes. The post multiplier introduced in Eq (2) can be justified by studying the case for which the gas is at a fixed adiabatic wall temperature, T_{aw} (or stagnation temperature, T_0). Then, for an isentropic mainstream, as the Mach number increases, the static temperature, T_e decreases, and the deposition rate is reduced as shown in Fig 4. In view of our above-mentioned discussion, specifying the adiabatic wall temperature implies specification of the thermophoretic suction effect, because the temperature gradient at the surface is thereby fixed which, in turn, fixes the thermophoretic velocity at the surface (even though a sufficiently high Ma_e can correspond to a mainstream cooler than the wall). Hence, any change in particle deposition rates (reduction) when Ma_e is varied can be attributed to the 'thermophoretic sink' operating outside the Brownian diffusion sublayer, as illustrated by the qualitative sketches given in Fig 5. The first panel shows the change in temperature profiles due to increase in Ma_e at constant T_w/T_{aw}. At first, Ma_e is not strong enough to produce a temperature maximum within the bl but, eventually, as Ma_e is increased further we observe the maximum as the mainstream gets cooler. The second panel in Fig 5 shows how V_T (directly proportional to $\partial T/\partial y$) would vary across the bl for the case $Ma_e = 0$ and for Ma_e large enough to produce a temperature maximum. Note that V_T becomes positive in the 'outer' region where the temperature gradient is negative, ie the particles are 'blown' away from the surface† although $V_{T,w}$ is the same for both $Ma_e = 0$ and $Ma_e > 0$. On the other hand, the pseudo-reaction rate constant, k, is approximately directly proportional to $\partial^2 T/\partial y^2$. As Ma_e increases, the curvature of the temperature profile also increases, which implies a larger k. However, in the region between the mainstream and the inflection point of the

† It is interesting to note that a similar 'reversal' of the sign of V_T occurs in low Mach number bls which contain a flame 'sheet', or (partially) distributed flame 'zone'

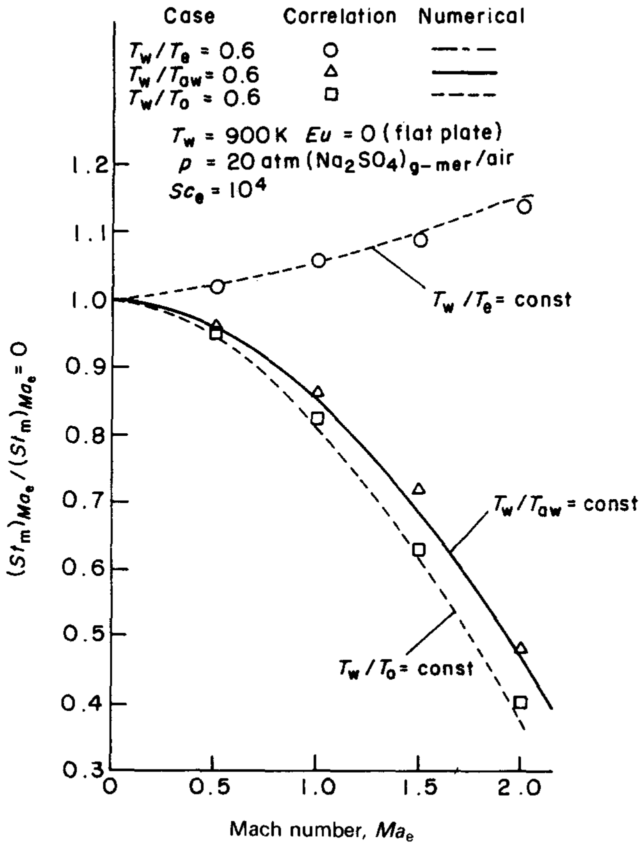

Fig 4 Effect of viscous dissipation on the thermophoretically enhanced mass transfer coefficient in a laminar boundary layer at constant mainstream, adiabatic wall and stagnation (reservoir) temperatures

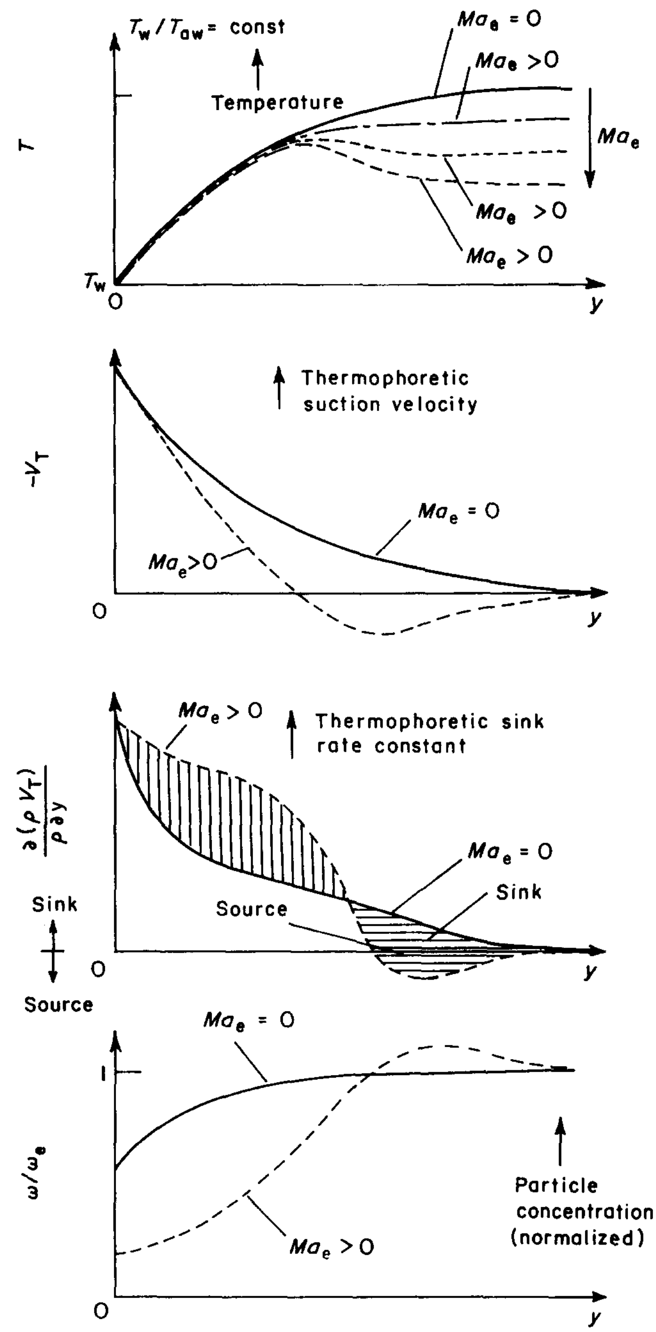

Fig 5 Effect of viscous dissipation on thermophoretic 'suction' and thermophoretic 'sink' effects at constant adiabatic wall temperature

temperature profile, k is negative, which implies a local 'source'. The overall effect, verified by our numerical calculations, is summarized qualitatively in the third panel, which shows an increased 'sink' effect in the presence of viscous dissipation. The net result of these phenomena is given in the last panel, where we show the corresponding concentration profiles, explaining why the deposition rate is reduced when Ma_e is increased for constant T_w/T_{aw} (or T_w/T_0). Although qualitatively the same, the quantitative difference between constant T_w/T_{aw} and T_w/T_0 cases in Fig 4 (which increases as Ma_e increases) is due to the nonunity recovery factor.

Conclusions

It has been demonstrated that, for a specified T_w and T_e, viscous dissipation within the momentum bl (associated with increased mainstream kinetic energy per unit mass) always increases the particle deposition rate, its effect depending very much on the T_w/T_e ratio. For combustion turbine blading which experiences near-sonic local bl 'edge' conditions, we estimated that viscous dissipation should increase particle deposition rates by about 25% at $T_w/T_e=0.8$ and about 5% at $T_w/T_e=0.6$, irrespective of particle diameter in the range: $0.6\times 10^{-2}\,\mu m \leqslant d_p \leqslant 0.3\,\mu m$.

For comparisons at constant T_{aw} or T_0 (reservoir temperature), on the other hand, increases in Ma_e (and their corresponding reduction in T_e) are associated with decreases in thermophoretically dominated particle deposition rates, the reduction being dependent on T_w/T_{aw} or T_w/T_0. However, for $T_w/T_{aw}=0.6$ and Ma_e near unity this reduction is only about 15%, irrespective of particle size (in the range $0.6\times 10^{-2}\,\mu m \leqslant d_p \leqslant 0.3\,\mu m$ at pressure levels of 20 atm). Thus, as conjectured in Ref 18, the primary (but not sole) effects of viscous dissipation are accounted for by replacing T_e by T_{aw} in the expression for the thermophoretic suction parameter, $-B_T$ (cf Eq (1)). As shown here, this procedure alone would overestimate particle mass transfer rates since it does not account for the reduction in ω_m/ω_e also caused by viscous dissipation.

The success of the engineering correlations of thermophoretically modified particle convective mass transfer reported earlier[19,20], including the effect of viscous dissipation, is demonstrated here for particle diameters up to about 0.3 μm at pressure levels of current turbine interest. These collective results should facilitate more accurate predictions of particulate mass transfer rates in severe forced convection environments encountered, say, in current and future combustion turbines. Indeed, we have recently demonstrated[21] that local (streamwise) application of these correlations, originally proposed and verified on the basis of theoretical considerations[18,19] and self-similar bl solutions[2,9,19], can provide acceptable predictions of St_m for nonsimilar (developing) boundary layers on turbine blades. However, further research (experimental and theoretical) remains to be one on other complicating effects (eg, fluid turbulence, particle inertia, wall roughness, film cooling, etc) on convective mass transfer coefficients, not considered here.

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